FAUP-JH-57-0059

J. H. FAUPEL and D. B. HARRIS

Engineering Research Laboratory, Engineering Department, E. I. du Pont de Nemours & Co., Inc., Wilmington, Del.

Stress Concentration in Heavy-Walled Cylindrical Pressure Vessels

Effect of Elliptic and Circular Side Holes

Experimental as well as theoretical information is given, in a field where most designers have had to make hopeful guesses

Since 1947 the Engineering Research Laboratory of the Du Pont company has been studying the mechanical behavior of materials and equipment under high pressure. A recently published article (*2*) on the behavior of heavy-walled cylinders under internal pressure contains design equations which predict the yield and bursting pressures of both plain and prestressed cylinders. The present article considers the effect of adding either elliptic or circular side holes to heavy-walled cylinders subjected to internal pressures (Figure 1).

A knowledge of the stresses at the cylinder bore–side hole interface is important, because many heavy-walled vessels contain oil holes for lubrication and ports for valves. In particular, for high-pressure applications, a realistic picture of the state of stress in a vessel with side ports is needed because fatigue life is very critical and present-day limitations of strength and ductility in commercial pressure vessel materials prevent high factors of safety. The information given here adds to knowledge of the behavior of vessels at high pressure; future revisions of pressure vessel codes may take advantage of these data. In work on thin-walled vessels (*1*, *6*), bending stresses induced by branch pipes and cover plates are considered. For heavy-walled vessels, two discussions (*3*, *4*) have considered briefly the stress concentration effect of a single circular side hole, but analytical procedures covering the general case (elliptic holes) or the case of cross-bore side holes have not been offered.

Analytical Procedure

In analyzing the state of stress in a cylinder containing side holes it is assumed that analyses established for holes in infinite elastic plates subjected to uniaxial or biaxial stresses can be used. For example, Figure 2 shows an elliptic hole in an infinite elastic plate subjected to tensile loading. For this case Wang has established (*8*), using conformal transformation, that

$$\sigma_x + \sigma_y = S[1 - m^2 - 2\cos 2(\beta - \theta) + 2m\cos 2\theta\cos 2(\beta - \theta) - 2m\sin 2\theta\sin 2(\beta - \theta)]/(1 + m^2 - 2m\cos 2\theta) \quad (1)$$

where

σ_x, σ_y = stresses in the X and Y cartesian directions
S = unit stress
m = $(a - b)/(a + b)$
a = semimajor axis of ellipse
b = semiminor axis of ellipse
β = angle between applied load direction and major axis
θ = angle defining any point on perimeter of ellipse with respect to major axis

When the loading direction is along the X axis ($\beta = 0$), the stresses at the ends of the ellipse axes are found as follows: At the ends of the minor axis $\theta = \pi/2$, $\sigma_y = 0$, and from Equation 1

$$\sigma_x = S_x\,[1 + 2\,b/a] \quad (2)$$

At the ends of the major axis, σ_x, β, and θ are all equal to zero and by Equation 1

$$\sigma_y = -S_x \quad (3)$$

When the loading is along the Y axis, the stress at the ends of the minor axis is found by letting β and θ equal $\pi/2$, with σ_y equal to zero in Equation 1; thus

$$\sigma_x = -S_y \quad (4)$$

and at the ends of the major axis, σ_x and θ equal zero and β equals $\pi/2$ in Equation 1; thus

$$\sigma_y = S_y\,[1 + 2\,a/b] \quad (5)$$

Thus, with appropriate values of σ_x, σ_y, β, and θ substituted in Equation 1, the state of stress for any biaxial loading condition can be determined for any point on the perimeter of the ellipse. Equation 1, for an elliptic hole in an infinite plate represents the general case; the circular hole is a special case of an ellipse with equal axes. Therefore, if the state of stress is required for a circular hole in a plate under uniaxial or biaxial loading, Equation 1 is modified by letting $a = b$. For a circular hole in a plate under uniaxial load, σ_x and σ_y in Equations 2 and 5 become equal to $3S_x$ and $3S_y$, respectively—or, a stress concentration factor of 3 is obtained. If the hole is not small with respect to the plate dimensions, other modifications are required; this is discussed later when the analysis is applied to the case of a side hole in a circular cylinder.

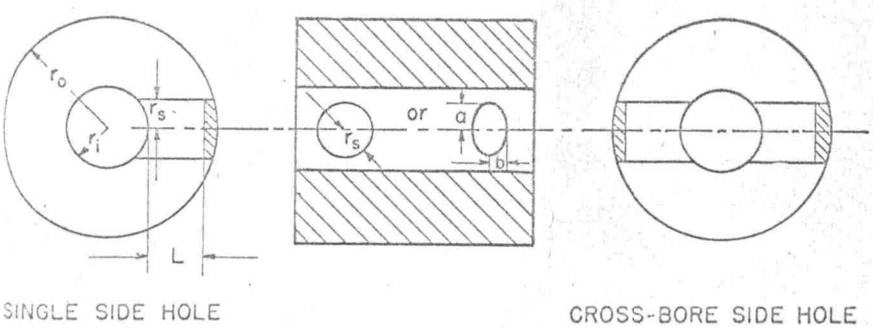

Figure 1. Geometry of side holes in cylinder

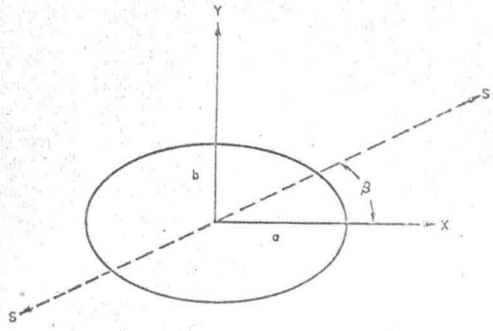

Figure 2. Elliptic hole subjected to unit stress S